Impatience vs. Incentives

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Impatience vs. Incentives

- **Incentives:** Dynamic (self-enforcing) contracts play an important role in many settings (Levin, 2003) such as models of
  - expropriation risk / sovereign debt
  - worker relationships
- **Impatience:** Agent impatience is common: (Acemoglu et al. (2008, 2010), Aguiar et al. (2009), DeMarzo Fishman (2007), etc.)
  - sometimes motivated by underlying economic situation:
    "Politicians are often argued to be more short-sighted than the agents, and the impact of political economy on intertemporal distortions is one of the questions motivating our analysis" (Acemoglu et al., 2008)
  - sometimes purely an auxiliary assumption

$\Rightarrow$ Incentives and impatience are prevalent features of economic models

Differential Discounting and Dynamic Agency

- Why is it of theoretical interest to study the interplay between impatience and incentives more generally?
- If the agent is impatient, both forces oppose each other!
  - Impatience: Agent should receive frontloaded rewards
  - Incentive provision: Agent payoffs should be backloaded
- Optimal resolution of this conflict is a priori unclear
- Main Result:
  Oscillation is generic feature of optimal dynamic contracts

Related Literature

- Incentive provision via backloading to agent (equal discounting)
  - Becker Stigler (1977), Harris Holmstrom (1982) and Ray (2002)
- Repeated games with *heterogeneous* discounting:
  - Lehrer Pauzner (1999) Dynamic trading gains imply frontloaded rewards to impatient player
- In our environment both incentives and impatience matter: Ray (2002) and Lehrer Pauzner (1999) can be understood as limits.

Framework

- Infinite horizon repeated interaction between principal P and agent A
  - discrete time
  - perfect public information
  - transferable utility
- Each period t, action a_t generates total suplus $\pi(a_t)$
  - agent receives total utility transfer $u_{A,t} \in \mathbb{R}$
  - principal receives $u_{P,t} = \pi(a_t) - u_{A,t}$
- Life-time (continuation) utilities $U_{i,t}$

$$U_{i,t} = \sum_{s=t}^{\infty} \delta_i^{s-t} u_{i,s} = u_{i,t} + \delta_i U_{i,t+1}$$

Agent is relatively impatient: $\delta_A < \delta_P$

Contracts, IC and PC constraints

Contract is stipulated sequence $\left\{\left(u_{A,t}, a_t\right)\right\}_{t=0}^{\infty}$ subject to:

- Incentive compatibility (**IC**) constraint of agent

$$U_{A,t} = \underbrace{u_{A,t} + \delta_A U_{A,t+1}}_{\text{Payoff within relationship}} \geq D\left(u_{A,t}, a_t\right) \forall t$$

D is payoff from *best possible* one period deviation given u_A and a

$$D\left(u_A, a\right) := \max_{a_D \in \Omega(u_A, a)} \check{D}\left(a_D \middle| u_A, a\right)$$

- Interim participation constraints (**PC**)

$$U_{A,t} \geq 0 \forall t$$
$$U_{P,t} \geq 0 \forall t$$

Example (Expropriation)

Setup: A government (A) allows a multinational firm (P) to invest I in the country. Investment generates output $Y(I)$ for the multinational and taxes τ. Then, $u_A = \tau$, and surplus $\pi = Y(I) - I$.

IC constraint of agent (government): The government can expropriate output up to $Y(I)$, but forfeits tax income forever.

$$U_{A,t} = \tau + U_{A,t+1} \geq D = Y(I) + \delta_A v_{aut}$$

- Observation: Higher transfers to agent relaxes IC one-by-one
- Related papers: Acemoglu et al. (2008), Aguiar et al. (2009), Opp (2012) and Thomas and Worrall (1994).

Example (Lending)

Setup: An entrepreneur (A) seeks a lender (P) to help finance a project. A project requires I from the lender and generates some return $Y(I)$ for the entrepreneur. The lender receives a stipulated loan repayment R. Then, $u_A = Y(I) - R$, and surplus $\pi = Y(I) - I$.

IC constraint of agent (entrepreneur): The entrepreneur can keep Y and strategically default on R, in which case the lender can take the entrepreneur to court. With probability $1 - \theta$ the lender prevails and recoups R. Otherwise he receives nothing.

$$U_{A,t} = Y(I) - R + \delta_A U_{A,t+1} \geq D = Y(I) - (1-\theta) R$$

- Observation: Lower R (higher $u_{A,t}$) *partially* relaxes IC by θ
- Related papers: Albuquerque Hopenhayn (2004)

Example (Shirking)

Setup: An owner (P) has access to a set of projects and can choose to collaborate with a worker (A) to implement a subset of them. Each subset $\{p_i\}$ requires effort cost $C_{A,\{p_i\}}$ from the worker, $C_{P,\{p_i\}}$ from the owner, and produces $\sum_{\{p_i\}} Y_{\{p_i\}}$ for the owner. The worker receives an up-front wage w. Then, $u_A = w - C_{A,\{p_i\}}$ and $\pi = \sum_{\{p_i\}} Y_{\{p_i\}} - C_{A,\{p_i\}} - C_{P,\{p_i\}}$.

IC constraint of agent (worker): The worker can shirk and keep the wage, in which case the worker is fired but captures unemployment benefits valued at $\delta_A O_A$.

$$U_{A,t} = w - C_{A,\{p_i\}} + \delta_A U_{A,t+1} \geq D = w + \delta_A O_A$$

- Observation: Higher wage does not affect IC
- Related papers: Thomas and Worrall (1988), Ray (2002)

Remark

If total deviation payoff D is quasilinear in the monetary transfer, then we can write incentive problem as:

$$U_{A,t} = u_{A,t} + \delta_A U_{A,t+1} \geq D(u_{A,t}, a_t) = (1-\theta) u_{A,t} + d(a_t)$$

or more compactly:

$$\theta u_{A,t} + \delta_A U_{A,t+1} \geq d(a_t)$$

- $0 \leq \theta < 1$ governs the sensitivity of IC to monetary transfers $u_{A,t}$
- $d(a)$ governs the severity of *IC* problem associated with action a

Example	θ	$d(a)$
Expropriation risk	1	Y
Lending example:	$\theta \in (0,1)$	θY
Worker	0	$c_{A\{p_i\}} + \delta_A O_A$

Problem

$$V\left(U_A\right) = \max_{\{(u_{A,t},a_t)\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \delta_P^t (\pi(a_t) - u_{A,t})\right] \; s.t.$$

$$u_{A,t} + \delta_A U_{A,t+1} \geq (1-\theta) u_{A,t} + d(a_t) \qquad \forall t$$

$$U_{i,t} \geq O_i \qquad \forall t$$

$$U_{A,0} \geq U_A$$

Assumption

i) *(π, d) maps action set $\mathscr{A}$ to $\mathbb{R}^2$ with compact image*

ii) *Outside options O_A, O_P sufficiently low*

Main Result

Theorem

There exists a unique steady state - a Pareto-optimal contract with a constant continuation payoff process $\{(U_{A,t}, U_{P,t})\}_{t=0}^{\infty}$. *The steady state action* a^s *does not maximize static surplus* $\pi(a)$:

$$a^s = \arg\max_{a \in \mathscr{A}} \pi(a) - \frac{\delta_P - \delta_A}{\theta(\delta_P - \delta_A) + \delta_A} d(a).$$

If $\theta = 0$ *or 1, every non-steady state Pareto-optimal contract has a continuation payoff process that converges monotonically to the steady state.*

If $\theta \in (0,1)$, *every non-steady state Pareto-optimal contract has a continuation payoff process that oscillates around the steady state. This oscillation persists in the long run if and only if* $\theta \in \left[\frac{\delta_A}{1+\delta_A}, \frac{\delta_A}{\delta_P+\delta_A}\right]$.

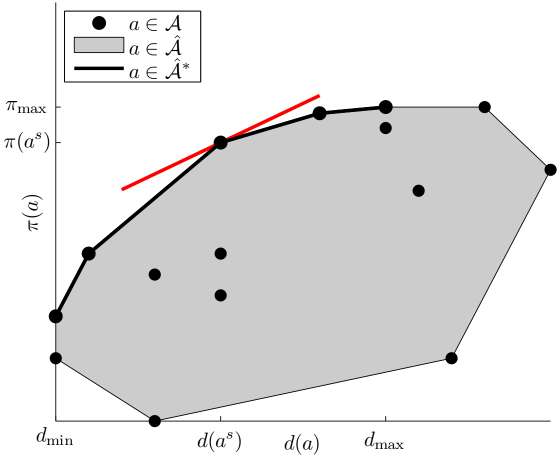
Intuition with simplest case

- Action set is a singleton $\{a^s\}$. Then, IC is $\theta u_{A,t} + \delta_A U_{A,t+1} \geq d(a_s)$
- Step 1: Relative impatience $\Rightarrow$ slack IC is generically suboptimal.
- Unique steady state transfer u_A^s defines $U_A^s = \frac{u_A^s}{1-\delta_A}$ and satisfies:

$$\theta u_A^s + \delta_A U_A^s = d(a_s)$$

- Dynamics of Pareto-optimal contracts with agent value $U_A \neq U_A^s$
  - Suppose $u_A > u_A^s$ and $U_{A,t+1} = U_A^s \Rightarrow$ IC constraint slack if $\theta > 0$!
  - Suppose $u_A < u_A^s$ and $U_{A,t+1} = U_A^s \Rightarrow$ IC constraint violated if $\theta > 0$!
- Relative impatience & $\theta > 0$ makes oscillation uniquely optimal!

General action set: Randomization and efficient actions



We can view scalar $d \in [d_{\min}, d_{\max}]$ as action and define $\pi(d)$

Efficient steady state action distortions

Lemma

$V(U_A)$ is a concave, strictly decreasing function over its domain $\left[U_A^{\min}, U_A^{\max}\right]$ and satisfies $V(U_A^{\max}) = O_P$.

Lemma

A Pareto-optimal stationary contract satisfies:

$$d^s \in \arg\max_{d \in \hat{A}^*} \pi(d) - \frac{\delta_P - \delta_A}{\theta(\delta_P - \delta_A) + \delta_A} d.$$

$$u_A^s = \frac{d^s}{\theta + \frac{\delta_A}{1-\delta_A}}.$$

In particular, d^s is generically smaller than $d_{\max}$.

Intuition for optimal investment distortions

$$d^s \in \arg\max_{d \in \hat{\mathcal{A}}^*} \pi(d) - \frac{\delta_P - \delta_A}{\theta(\delta_P - \delta_A) + \delta_A} d$$

1. Start off with homogeneous discounting $\Rightarrow d^s = d^{\max}$ & $\pi^s = \pi^{\max}$
2. Increase discount factor of principal: $d^{\max}$ still feasible, but
   1. Moving payments from future to today creates a first-order dynamic trading gain (but hurts incentives)
   2. Distorting action at $d^{\max}$ creates second-order loss of static surplus π
3. Optimal stationary action choice d^s trades off both sources of surplus
4. Degree of distortion depends on dynamic trading gains $\delta_P - \delta_A$

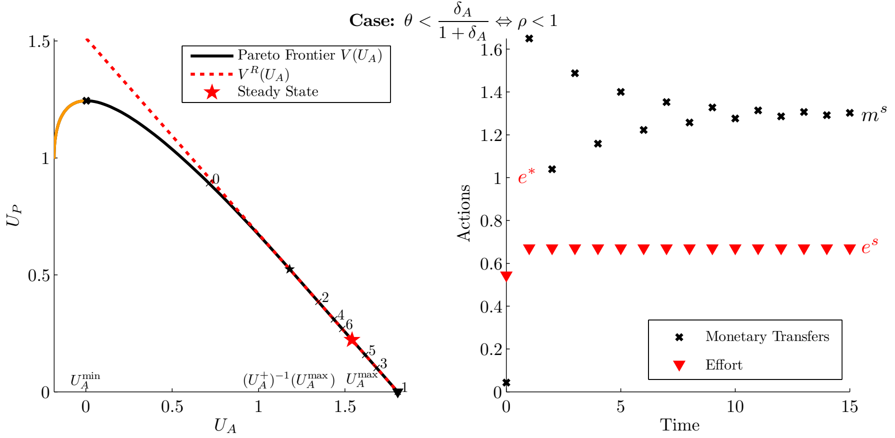
Pareto-optimal non-stationary contracts

- Benchmark contract: $d = d^s$ and oscillating transfers such that IC binds at ALL times (as in singleton case). Then:

$$U_A^+ (U_A) - U_A^s = -\underbrace{\frac{\theta}{(1-\theta)\,\delta_A}}_{\rho} (U_A - U_A^s)$$

- For $\theta \in [0, \frac{\delta_A}{1+\delta_A}]$
  - Benchmark contract does not explode $\rho \leq 1$
  - Pareto frontier is linear through steady state and hence unimprovable
  - Participation constraints (may) only play a role at time 0
- For $\theta > \frac{\delta_A}{1+\delta_A}$
  - Any non-stationary benchmark contract would eventually violate PC
  - Pareto frontier no longer linear
  - Action will generally not be held constant

Dynamics Case 1:



Worker example with quadratic effort cost and $\theta \in \left(0, \frac{\delta_A}{1+\delta_A}\right)$

Action distortions when benchmark contracts explode

- Say $U_{A,0} = U_A^{\max}$ and principal wants to use benchmark contract
  - Contract will violate participation constraint of P in two periods!
  - To respect PC of principal, the agent's date 2 payoff must be lowered
  - But this violates the IC constraint of agent at date 1
  - To maintain IC either decrease action $d_1 < d^s$ or increase $U_{A,1}$

$$U_{A,1} = U_A^+(U_A^{\max}) > U_A^s - \rho\left(U_A^{\max} - U_A^s\right)$$

  - Intuitively do both! $\Rightarrow$ But now the date 0 IC constraint is slack
  - Principal can increase the date 0 action $d_0 > d^s$
- Intuition: action distortions dampen oscillation (overshooting)
  - Relax incentive problem (and lower surplus) when $U_A < U_A^s$
  - Tighten incentive problem (and increase surplus) when $U_A > U_A^s$

Exact dynamics of action distortions

Assumption

$\pi(d)$ is a strictly concave, continuously differentiable function tracing out slopes $\pi'(d) \in \left[0, \frac{1}{\theta}\right]$.

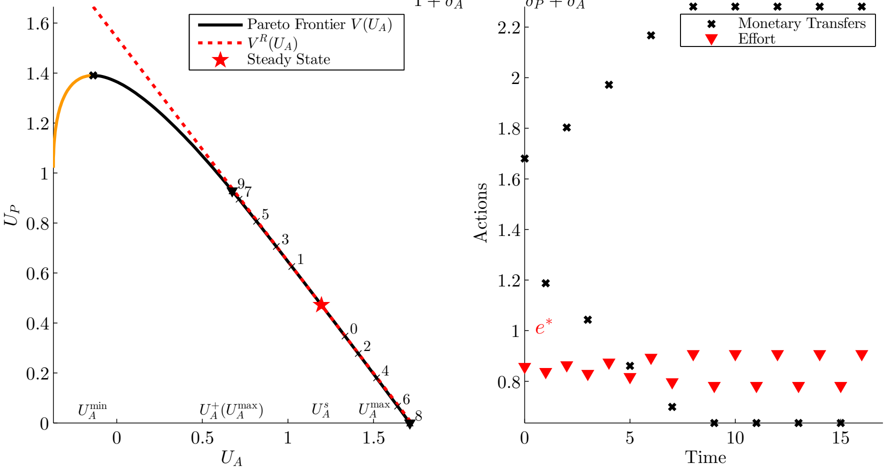
- Then, we obtain exact characterization of action dynamics:

$$\delta_P \underbrace{\left[\pi'(d^+) - \pi'(d^s)\right]}_{\text{Marginal Distortion tomorrow}} = -\frac{1}{\rho} \underbrace{\left[\pi'(d) - \pi'(d^s)\right]}_{\text{Marginal Distortion today}}$$

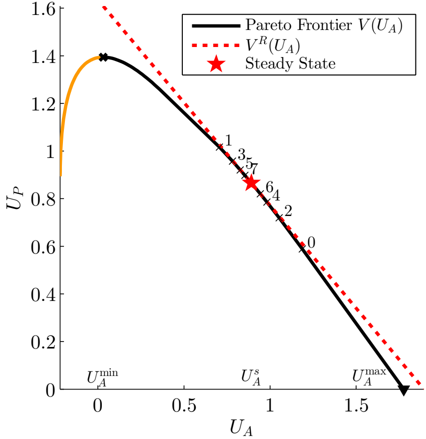
  - Tomorrow's distortion is naturally discounted by δ_P while
  - Adjustments today are stronger the higher θ
- Distinguish two subcases
  - If $1 < \rho < \frac{1}{\delta_P}$ benchmark contracts and actions explode (Case 2)
  - If $\rho > \frac{1}{\delta_P}$ actions converge (Case 3)

Dynamics Case 2

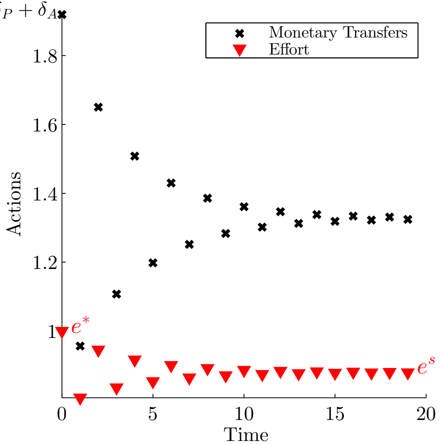
Case: $\frac{\delta_A}{1+\delta_A} < \theta < \frac{\delta_A}{\delta_P+\delta_A}$



Dynamics Case 3



$\theta > \frac{\delta_A}{\delta_P + \delta_A}$



Ray vs. Lehrer-Pauzner

- We assumed that neither PC constraint binds in steady state
- Otherwise, the steady state is at a corner of V
  - no room to oscillate around steady state
  - V-contracts monotonically converge
- Formalization: Consider shifting deviation payoff $d(a) + x$ for $x \in \mathbb{R}$
  - Larger x: incentive problem getting stronger
  - Smaller x: incentive problem getting weaker

Ray vs. Lehrer-Pauzner

Corollary

i) *When the impatience force dominates the incentives force* ($x \leq \underline{x}$)*, the steady state is the leftmost V-contract and all other V-contracts are frontloaded, monotonically converging leftwards to the steady state.*

ii) *When the incentives force dominates the impatience force* ($x \geq \overline{x}$)*, the steady state is the rightmost V-contract and all other V-contracts are backloaded, monotonically converging rightwards to the steady state.*

iii) *When neither force dominates* ($\underline{x} < x < \overline{x}$)*, the impatience vs incentives conflict is nontrivial, and oscillation around the steady state is a generic feature of V-contracts.*

Robustness

- Analysis identifies two forces that make oscillation uniquely optimal
  1. Relative impatience introduces dynamic trading gains $\Rightarrow$ IC binds
  2. Pay today provides additional incentives today $(\theta > 0)$
- Robustness of mechanism to
  - limits on monetary transfers
  - θ does not need to be linear function of monetary transfer
- Conjecture: mechanism also robust to (sufficiently weak) principal IC constraint! $\Rightarrow$ allows us to study partnerships!

Positive (negative) Implications

- Oscillation is a natural feature of efficient contracts
  - Efficient contracts try to tap "dynamic trading gains" as well as static relationship gains (investment)
  - Subtle role of participation constraints
- How to interpret our theoretical results?
  - Is impatience motivated by the underlyingeconomic environment?
    $\Rightarrow$ positive prediction about real-life phenomena (cycles!)
  - or just an auxiliary assumption?
    $\Rightarrow$ negative result: strong side-effects